Towards a Probabilistic Roadmap for Multi-robot Coordination

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Abstract—*In this paper, we discuss the problem of multi-robot coordination and propose an approach for coordinated multi-robot motion planning by using a probabilistic roadmap (PRM) based on adaptive cross sampling (ACS). The proposed approach, called ACS-PRM, is a sampling-based method and consists of three steps including C-space sampling, roadmap building and motion planning. In contrast to previous approaches, our approach is designed to plan separate kinematic paths for multiple robots to minimize the problem of congestion and collision in an effective way so as to improve the system efficiency. Our approach has been implemented and evaluated in simulation. The experimental results demonstrate the total planning time can be obviously reduced by our ACS-PRM approach compared with previous approaches.*

Keywords: Multi-robot system; motion planning; multi-robot coordination; sampling-based approach;

1. Introduction

Motion planning is a fundamental problem in robotics. It could be explained as producing a continuous motion for a robot, that connects a start configuration and a goal configuration, and avoid collision with any static obstacles or other robots in an environment. The robot and obstacle geometry are generally described in a 2D or 3D workspace, and the motion could be represented as a path in configuration space. Motion planning algorithms are widely applied in many fields, such as bioinformatics, robotic surgery, industrial automation, planetary exploration, and intelligent transportation system.

The multi-robot system (MRS) is proposed to deal with some problems that are difficult or impossible to be solved by a single robot, or to improve the system implementation efficiency in some missions completed by multi-robot rather than a single robot [1], [2]. The biggest challenge for the MRS is coordination. Without coordination, it will not only lower the system efficiency, but also lead to the failure of the entire system in extreme cases. Figure 1 shows an example of multi-robot coordination, four robots implement a transportation mission cooperatively, the red robot is delivering a goods, the green robot is on its way back after completing a transportation task, the yellow robot is moving to load a goods, and the orange robot is transporting a goods to the destination location. The coordination of these four robots is obtained by assigning them to different room.

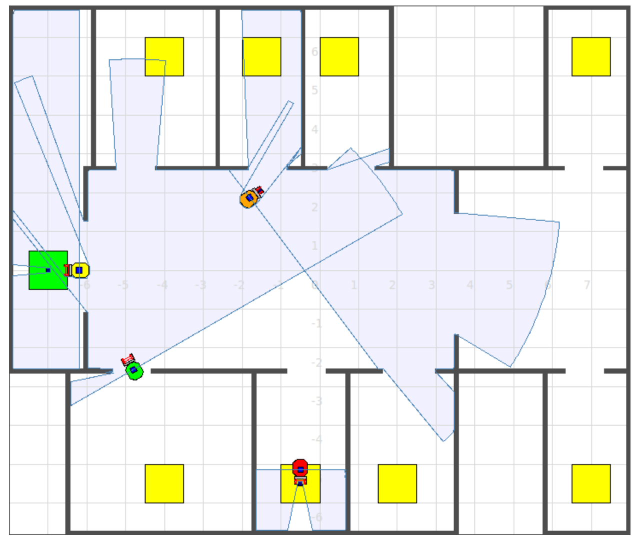


Fig. 1: Four robots implement a transportation mission cooperatively. The dark blue piece signifies the goods to be transported. The green area represents the original position of goods, and the yellow area represents the destination, which corresponds to every room, where the goods should be delivered to by the mobile robot.

In this paper, we consider the issue of coordinated motion planning for a homogeneous team of autonomous mobile robots in structured environments such as office building, warehouse, and container terminal. The larger context of the research is to establish a multi-robot goods transportation system with security, reliability and efficiency. Most of the proposed approaches for multi-robot motion planning usually have the problem of resource conflict such as congestion and collision [3], [4]. For the transportation issue, a desirable result is that robots replan their individual local path to avoid collision and congestion events, however this fashion often needs additional time and thus limit the transportation efficiency. An undesirable result is that robots are blocked, or the goods are lost or damaged, and thus fail the transportation mission. Therefore, we arranged these cases to the waiting situation problem [5]. To handle this practical problem, this paper presents a novel approach to multi-robot motion planning by using a probabilistic roadmap (PRM) planner which is based on manner of adaptive cross sampling (ACS). This approach called ACS-PRM is decomposed into 3 main steps:

- Firstly, a sufficient number of points should be generated in C-space on an occupancy grid map by using an adaptive cross sampling method.
- Secondly, a roadmap should be built while the potential targets and milestones are extracted by post-processing the result of sampling.
- Finally, the motion of robots should be planned by querying the constructed roadmap.

The rest of the paper is organized as follows: Section 2 describes an overview of some related works; Section 3 discusses the problem of waiting situation; subsequently, Section 4 describes our ACS-PRM approach; Section 5 presents the experimental results obtained with our approach; and the paper is concluded in Section 6 at last.

2. Related Work

Multi-robot motion planning has been extensively studied for more than a decade during which a wide variety of planning frameworks and solutions have been proposed.

Švestka and Overmars [6] presented an approach for multiple nonholonomic car-like robots motion planning in the same static workspace by using probabilistic roadmaps, in which the roadmaps for the composite robot are derived from roadmaps for the underlying simple robots, and the latter is computed by a probabilistic single-robot learning method. The authors introduced the notion of *super-graphs* for multi-robot path planning, and their implementation covered the construction of the simple roadmap and the super-graphs. This approach is probabilistically complete because a given problem could be solved within a finite amount of time.

Moors *et al.* [7] presented a graph-based algorithm for coordinate multi-robot motion planning in 2D indoor environments. The scenario of this research is multi-robot indoor surveillance. The proposed approach takes the limitations and uncertainties of sensors into account, and generates the coordinated motion plan for multiple robots by using A* search algorithm. The authors also introduced a framework based on realistic probabilistic sensor models and worst case assumptions on the intruder's motions in order to compare different approaches and evaluate the coordination performance of the proposed approach.

Clark [8] presented a multi-robot motion planning strategy based on the probabilistic roadmap within a dynamic robot network (DRN) coordination platform. The DRN platform is an ad hoc network, in which single-query PRM is queried as a centralized planner to plan trajectories for all robots. The PRM planner is optimized to speed queries for multi-robot motion planning by using new sampling strategies. At first, a method of sampling PRM milestones is identified to enable fast coverage of the configuration space. Then, a method of generating PRM milestones is introduced to decrease the planning time. Finally, an endgame region is defined to improve the likelihood of finding solutions when goal configurations are highly constrained.

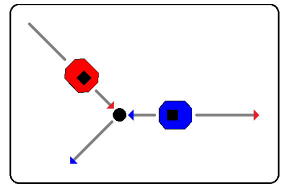


Fig. 2: A typical waypoint mutex. The black dot represents the waypoint, the gray segment represents the path, and the red and blue arrow represent the direction of the motion of the corresponding color robots respectively.

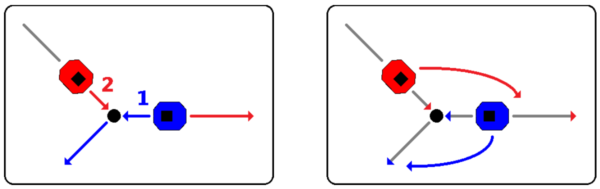
Saha and Isto [9] presented a strategy for decoupled multi-robot motion planning. The proposed strategy, which aims at improving the reliability of the basic decoupled planning approach, partially merges the two phases of the basic approach. The first phase is to compute a collision-free path to avoid the obstacles in the environment and the other robots, the second phase is coordinating the individual robot motions so that only one robot at a time may enter the area of potential inter-robot interference. The proposed approach searches for motions for a robot and coordination of motions of robots along paths already planned while ignoring the robots whose motions have not been planned simultaneously. This approach is inherently incomplete.

Besides, there are some other approaches developed with various strategies [10], [3], [11], [4].

3. Waiting Situation Problem

Multi-agent environments can be cooperative or competitive [12]. Of every agent in a team, the other agents can be considered as teammates (cooperative) or movable obstacles (competitive). One of the most important reasons which limit the efficiency of the multi-robot motion planning is the waiting situation such as the congestion and collision between robots. The core of the problem can be considered as the waypoint mutex in multi-robot motion planning. Figure 2 depicts a typical waypoint mutex. Two robots move to the same waypoint simultaneously: the red robot moves from the top left towards the right and the blue robot moves from the right towards the bottom left. Because of the waypoint can be assigned only to one robot at a time, then the mutex of the waypoint happens.

Generally, there are two ways to deal with the waypoint mutex as shown in Figure 3. One (Figure 3(a)) is to let robots pass the waypoint one by one [6]. The weakness of this strategy is that one robot must wait for another robot to pass. Another way (Figure 3(b)) is to replan the



(a) (b)

Fig. 3: Two ways to deal with the waypoint mutex. The colored lines represent the motion plan for the corresponding color robots respectively. (a) The two robots pass the waypoint in order, the blue robot pass first and the red robot pass later. (b) The two robots take each other as an obstacle and replan its trajectories in real time.

local path in real time for each robot by using some goal seeking obstacle avoidance algorithms such as Vector Field Histogram (VFH+) [13] or Nearness Diagram (ND) Navigation [14]. The weakness of this strategy is that robots need some time to replan their new trajectory. Consequently, the two ways both extend the time of the motion planning and limit the system efficiency.

If we can plan separate kinematic paths for multiple robots, then the waiting situation caused by waypoint mutex will be obviously reduced. In addition, the problem of multi-robot task allocation (MRTA) [15] should also be considered. Our focus in this paper is on the multi-robot motion planning in structured environments. For instance, an office building could be simply divided into three clusters: corridor, doorway and room. The doorway usually connects the corridor with the room, it is a suitable object for task allocation. Therefore, on the one hand, the problem of waiting situation (especially at the corridors) caused by one path for all robots should be solved. On the other hand, a simple and effective way to coordinate multi-robot motion is to assign different robots to different rooms reachable from the corridors. Besides, more complex environments may require more sophisticated methods such as hand labeled training data [16], [17] or more complex reasoning [18], [19].

Because of the complexity of multi-robot systems [10], [20], the target of this paper is not to completely avoid the problem of waiting situation (i.e., waypoint mutex), but to minimize the probability of appearance of the waiting situation by using our ACS-PRM approach. Therefore, the coincidence of the waypoint in the plan of two or more robots should be reduced so as to improve the multi-robot motion planning efficiency.

4. ACS-PRM: Adaptive Cross Sampling Based Probabilistic Roadmap

There are usually two ways to handle the issue of robot motion planning based on the grid representation of the environment in low dimensional space. One is to use the incremental heuristic search algorithm such as A* [21] or D* [22]. Another is to use the topological map [23] generated on top of the grid-based map such as Voronoi diagram and straight skeleton. Nevertheless, the number of grids increases rapidly when the size of the environment expands, which make these methods inappropriate for complex and extensive environments. Moreover, these methods are hard to deal with the multi-robot motion planning and always increase the computational load.

Sampling-based approaches have been proposed to improve the computational efficiency for robot motion planning. The main idea is to avoid the explicit construction of the obstacle region in the C-space (C_{obs}). Unlike the incremental heuristic search and the topological map methods, the sampling-based approaches work well for complex environments and high-dimensional configuration spaces, and they are generally easier to implement. The probabilistic roadmap (PRM) planner[1] is one of the typical sampling-based approaches. The original PRM technique is introduced by Kavraki *et al.* [24], which has been shown to perform well in a variety of situations. On the basis of this method, different extensions have been proposed [25], [26], [27]. The approach described in this paper is also an extension of PRM, which is aimed at performing multi-robot motion planning efficiently.

To deal with the problem of waiting situation in multi-robot motion planning as mentioned in the last section by using the PRM approach, there are substantially two options:

- In the manner of single-query: when two or more robots need to pass the same waypoint simultaneously, each robot resamples the adjacent region and takes the motion of the others into account, then generate a novel local roadmap for local replanning.
- In the manner of multi-query: construct a rich roadmap at the beginning to allow robots to plan a different trajectory from others later.

4.1 C-space Sampling

The ACS-PRM approach presented in this paper is a multi-query approach. The first step is C-space sampling, in which a sufficient number of points should be generated to represent the free space of the environment. The main idea of this step is to let a random point p retracts to a position $P(q)$ with the distance d to the obstacle C_{obs} along horizontal and vertical directions (i.e., cross direction).

For autonomous nonholonomic mobile robots, in two dimensions, there are three representational degrees of freedom (DOFs) which are one rotational DOF and two translational DOFs (along or across), but only two controllable DOFs which only move by a forward motion and a steering

[1]A reference implementation of this method in C++ is available online at: http://www.ai.univ-paris8.fr/ yz

angle, the configuration space C is the special Euclidean group $SE(2) = \mathbb{R}^2 \times SO(2)$ where $SO(2)$ is the special orthogonal group of 2D rotations. To avoid the collision caused by the point retracts too close to the obstacle, we set the distance d as the sum of the positive number w and the radius r of the minimum circle to cover the robot with centering at the rotation center of the robot:

$$d = r + w, \ (w > 0) \tag{1}$$

The set of w is to deal with the negative influence of sensor error and it should be adjusted in practical applications.

C_{obs} represents the set of the obstacle, $\forall q \in C_{obs}$ define a direction r_q, then determine a symmetry point $S(q)$ which is an intersection of the open-ray with end q direction r_q and another C_{obs}:

$$S(q) = \{q + t\overrightarrow{r_q} | t > 0\} \cap C_{obs} \tag{2}$$

where, if $\{q + t\overrightarrow{r_q} | t > 0\} \cap C_{obs} = \{q\}$, then define $S(q) = \infty$. Let $dist(x, y)$ represent the distance between point x and point y, then the retraction function can be described as:

$$P(q) = \begin{cases} q + d\overrightarrow{r_q} & \text{if } dist(q, S(q)) \geq 2d \\ \frac{q+S(q)}{2} & \text{otherwise} \end{cases} \tag{3}$$

where $P(q)$ is the position for the point p to retract. In this way, the random points are adapted around to the obstacle (see Figure 4(b)), then:

$$\text{ACS-PRM} = \{P(q)\} | q \in C_{obs}\} \tag{4}$$

The implementation of this step is summarized in Algorithm 1, where the time complexity is $O(n)$ and the space complexity is $O(1)$. This step corresponds the learning phase of classic implementation of PRM.

4.2 Roadmap Building

The second step is roadmap building, in which the potential targets and the milestones should be extracted and connected to the roadmap. In the previous step C-space sampling, if there are sufficient points generated, then the points will gather into segments. The main idea of this step is post-processing the graph resulted from the previous step while identifying three types of point as follows:

- In the previous step, if $dist(p, q) < d$, then p will retract to $\frac{q+S(q)}{2}$ and be labeled as the medial axis. Therefore, we find those medial axis segments with length l a small fixed value (in our implementation, we took the thickness of obstacle), and the midpoints of segments are marked as *potential target* for task allocation. Figure 4(c) shows the extracted potential targets which are precisely doorways of the structured environment.

Algorithm 1 Adaptive cross sampling

Require: N, the sufficient number of points to generate.
Ensure: N points in C_{free} by adaptive cross sampling.

repeat
  Generate a uniformly random point p in C-space.
  if p is free **then**
    for horizontal and vertical directions **do**
      Find $q \in C_{obs}$ the nearest distance from p.
      if $dist(p, q) \geq d$ **then**
        p retracts to $q + d\overrightarrow{r_q}$.
      else
        Find $\{S(q)\} = \overrightarrow{qp} \cap C_{obs}$.
        p retracts to $\frac{q+S(q)}{2}$.
      end if
    end for
  end if
until N points have been generated.

- For those segments without containing the potential target, we extract both of the endpoints and mark them as *milestone* (see Figure 4(d)).
- The points of intersection between two segments are also extracted and marked as *milestone* (see Figure 4(d)). These milestones have not been used in our experiments, but they will be required for the exploration problem.

This step also corresponds the learning phase of classic implementation of PRM. Figure 4 illustrates the process of generating a roadmap for an example occupancy grid map by using our approach with 200,000 random samples.

4.3 Motion Planning

The third step is motion planning, in which each individual robot's kinematic path should be planned by querying the constructed roadmap. The main idea of this step includes the following three points:

- The potential targets $\{t_i\}$ are considered as the goal nodes for path planning and the objects for task allocation as well. Then, the individual $\{r_i\}$ robots are assigned to different potential target:

$$\{r_i\} \mapsto \{t_i\} \tag{5}$$

- To maximize the difference between the paths, we assign the potential target which is the closest from the robot but further from the previously assigned target to the current individual robot:

$$t = further(closest(\{t_i\}, r), t_{i-1}) \tag{6}$$

- Similar to the classic PRM, we use the fast local planning method (i.e., the straight line planner) for the global path planning, except that we choose the path

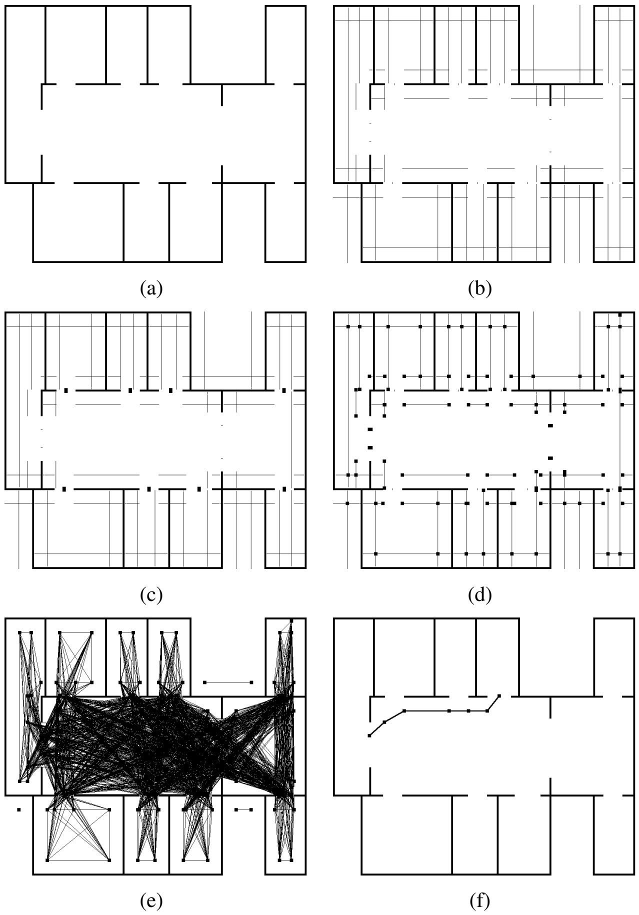
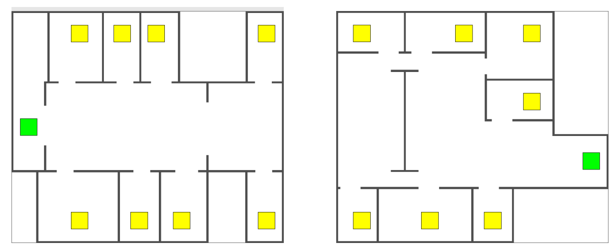


Fig. 4: Generation of the roadmap based on ACS-PRM. (a) The original gridmap, (b) adaptive cross sampling in C-space, (c) the extracted potential targets (doorways), (d) the extracted milestones, (e) the roadmap coverage of environment, and (f) an instance of path.

Fig. 5: A typical prototype of Pioneer 2-DX robot with gripper.



(a) map A

(b) map B

Fig. 6: Two environment maps used in our simulation.

with the minimum number of milestones for the robots invariably.

This step corresponds the query phase of classic implementation of PRM.

5. Experiments

To evaluate our ACS-PRM approach, we conducted a series of simulation experiments with the well-known 2D multi-robot simulator Stage [28]. The experiment is to transport a certain amount of goods from one origination to divers destinations by a fleet of mobile robots. The simulated robot is the Pioneer 2-DX robot equipped with a laser range finder providing 361 samples with 180 degrees field of view and a maximum range of 8 meters. Each robot can localize itself based on an abstract localization device which models the implementation of GPS or SLAM. To transport goods, the robots are equipped with a gripper that enable them to sense, pick up and put down the goods, and the carrying capacity is limited to one unit per robot. Figure 5 shows a typical prototype of Pioneer 2-DX robot with gripper.

We used a different number of robots to conduct several experiments in various environments. Two maps (Figure 6) were used in our simulation which are both structured environments. For each map, the green area signifies the original position of goods, and the yellow areas represent the destinations which are always placed in the rooms. For instance, map A has 8 rooms thus 8 destinations, map B has 7 rooms thus 7 destinations. The transportation team size is varied from 2 to 8 robots. On each team size, 10 experimental runs are performed for a transportation mission of 50 goods. The mission objective is to transport the goods to every room equally.

The ratio between real-world time and simulation time is about 1:1. We also compared our approach to the commonly used Voronoi-based approach [23] in which a topological map is built on top of the grid map by using the Voronoi diagram, and the *critical points* are extracted like milestones for mobile robot motion planning. All experiments reported in this paper were carried out on a system with an Intel Core 2 Duo E8400 3.00GHz processor, an Intel Q43 Express chipset and two DDR2 800MHz 1024MB dual channel memory.

In the experiments, we assumed that there exists a central server which is able to communicate with all mobile robots and assign the transportation tasks to each individual robot. The transportation task is to transport the goods from the original position to the destination. We also assumed that all the mobile robots share a common grid map and everyone

Table 1: Statistics of The Number of Occurrences of The Waypoint Mutex

(a) map A

#robots	2	3	4	5	6	7	8
ACS-PRM	1.6	3.1	6.4	7.5	10.0	11.1	16.2
Voronoi-based	15.3	18.7	26.0	26.8	19.9	23.7	27.2

(b) map B

#robots	2	3	4	5	6	7	8
ACS-PRM	3.8	4.3	7.1	14.9	12.8	10.3	16.7
Voronoi-based	17.1	19.2	19.0	26.5	27.4	27.0	29.9

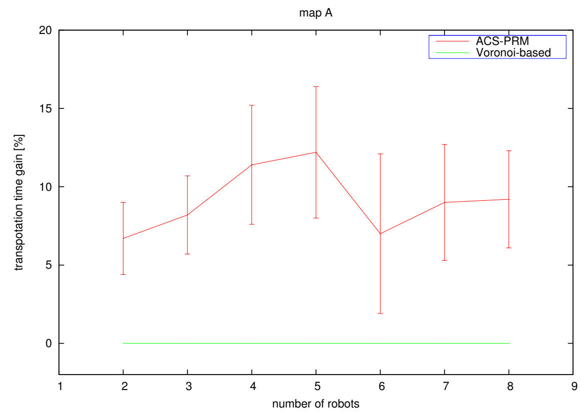
has full information about all others so as to implement path planning and obstacle avoidance in real time coordinately. The ACS-PRM is designed to (but not limited to) plan the kinematic path for nonholonomic mobile robots, and in order to get an objective evaluation of the proposed approach, the drive mode of mobile robot is set to differential-steer, furthermore, the strategy of one pass after the other is applied to deal with the possible waypoint mutex problem.

The results of our experiments are given in Figure 7. We measured the transportation time gained by our approach and compared to the Voronoi-based approach. In each plot, the abscissa denotes the team size of the mobile robots, the ordinate denotes the percentage of the transportation time in the total transportation time, and the error bar indicates the confidence interval of each corresponding gain of robot team size with the 0.95 confidence level. Figure 7 shows that, a transportation time saving of 6.7% to 12.2% in map A and 6.1% to 12.0% in map B is obtainable under our ACS-PRM approach compared to the Voronoi-based approach. These results proved that our technique could obviously improve the system of planning efficiency.

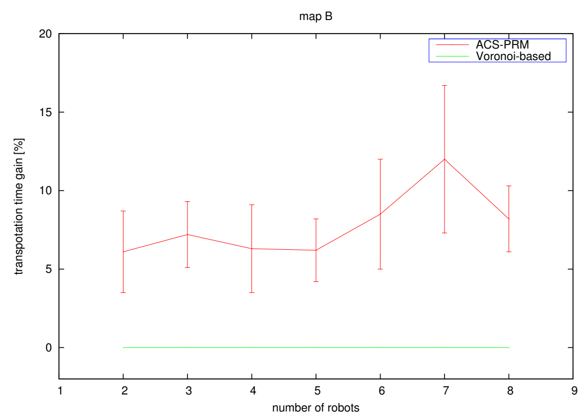
Moreover, we mentioned earlier that our ACS-PRM approach is more effective than our previous approach because the ACS-PRM spends much less time for the learning phase. The experiments show that, with the new approach, the mapping times are respectively 0.321 seconds and 0.329 seconds for map A and map B with 200,000 random samples, which are averages of the 10 runs.

We also counted the average number of occurrences of waypoint mutex in each map as shown in Table 1. This table shows that the problem of waiting situation is obviously reduced by using our ACS-PRM approach, because our approach is able to plan separate paths for robots, especially in the corridor. Unlike the Voronoi-based approach, there is only one path for all robots.

In fact, the technique proposed in this paper also works well with irregular environments. Figure 8 illustrates an example with 20,000 random samples.

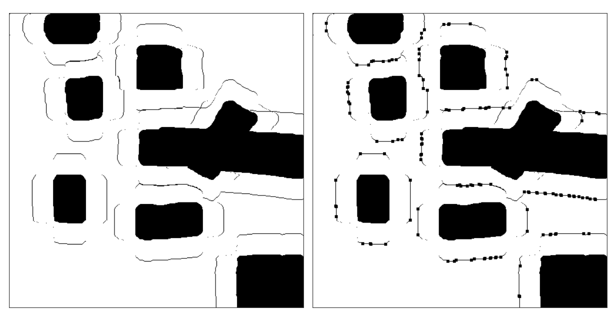


(a) map A



(b) map B

Fig. 7: Transportation time gained by using our ACS-PRM approach compared with the Voronoi-based approach.



(a) (b)

Fig. 8: Irregular environment experiment based on ACS-PRM. (a)Adaptive cross sampling in C-space, (b) the extracted milestones.

6. Conclusion

In this paper, we presented a novel approach for coordinated motion planning of multiple robots by using the probabilistic roadmap planner based on a manner of adaptive cross sampling, which we called ACS-PRM. The basic thought of the proposed approach is to build separate kinematic paths for multiple robots to minimize the problem of waiting situation such as collision and congestion caused by waypoint mutex in an effective way, thus to improve the efficiency of automated planning and scheduling.

The ACS-PRM mainly consists of three steps: C-space sampling, roadmap building and motion planning. In the first step, a sufficient number of points are generated to represent the free space of the environment. In the second step, the potential targets and the milestones are extracted and connected to the roadmap by post-processing the graph resulted from the previous step. In the third step, the robot's motion planning is done by querying the constructed roadmap. The first two steps correspond the learning phase of classic implementation of PRM, and the last step corresponds the query phase of classic implementation of PRM.

In consideration of the context of the issue of multi-robot goods transportation, the experiments were conducted to transport a certain amount of goods by a fleet of mobile robots in structured environments. The experimental results demonstrate that, by using our ACS-PRM approach, the total time needed to complete the transportation mission has been obviously reduced compared to the Voronoi-based approach.

In our future work, we will expand our experiments to various irregular environments, not just the structured environments. Furthermore, the proposed work in this paper can be also used in some other applications such as exploration mission, automated surveillance, and search and rescue operations. They are our future consideration as well.

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